Numerical Modeling of Helium-II in Forced Flow Conditions

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A numerical model was developed to investigate the forced flow behavior of liquid helium-II (He II) in a large duct ($d > 1$ cm). The model solved the conservation equations associated with the two-fluid formulation of He II using a finite-difference method. The transport terms in these equations were formulated in a similar way as for a classical Newtonian fluid. Predicted liquid temperature and pressure drop agreed well with data obtained from laboratory experiments. At transfer rates above 0.5 m^3/h ($Re > 1.5 \times 10^6$), the model predicted that forced convection would dominate over the Gorter-Mellink conduction in He II heat transfer, and that the classical pressure drop correlation for turbulent flow could adequately analyze the flow. The model can be used for designing spaceborne He II resupply systems.

Nomenclature

A_{GM} = Gorter-Mellink coefficient
C_f = Darcy friction coefficient
C_p = specific heat
D = diameter of transfer line
f = Fanning friction coefficient
F_{mf} = mutual friction force
H = enthalpy
K = Gorter–Mellink conductance parameter
L = length of transfer line
$\dot{m}$ = helium mass flow rate
P = pressure
ΔP_2 = pressure drop calculated using two-fluid model
ΔP_c = pressure drop calculated using classical turbulent flow
q = parasitic heat load on transfer line per unit length
q_H = heat flux from heater
q_0 = discrete heat source on transfer line
q'' = parasitic heat load on transfer line per unit area
q''' = parasitic heat load on transfer line per unit volume
Q = helium volumetric flow rate
Re = Reynolds number
S = specific entropy
T_0 = helium temperature at entrance of transfer line
T_L = helium temperature at exit of transfer line
U = velocity of He II
z = axial coordinate
z_0 = locations of discrete heat sources on transfer line
δ = Dirac delta function
ρ = density
μ = viscosity
τ''' = shear stress per unit volume

Subscripts and Superscripts

j = jth node
n = normal component of He II
s = superfluid component of He II
w = wall

Introduction

TRADITIONALLY, prediction of helium-II (He II) flow phenomena has been a domain of theoretical physics. Glaberson and Schwarz[1] employed the concepts of quantum vortices to perform a microscopic numerical modeling of He II. The model required adjustable parameters to fit turbulent flow test data. Recently, studies of large-scale spaceborne systems [such as the space infrared telescope facility (SIRTF) and the particle astrophysics magnet facility (ASTROMAG)] showed that on-orbit replenishment of He II may substantially extend the mission lifetime. Consequently, on-orbit He II transfer, which typically requires large ducts ($d > 1$ cm), stimulates the effort of analyzing the fluid flow from a macroscopic point of view. This paper discusses the macroscopic modeling of turbulent He II flow inside a large transfer line.

The physical behavior of He II is, so far, best described by the two-fluid model,[2,3] which postulates that He II behaves as if it were a mixture of two interpenetrating fluids, superfluid and normal fluid. The two fluid components have different physical behavior and properties. For example, superfluid has no viscosity and carries no entropy. Note that the two-fluid representation is only a model and the existence of the two components is a hypothesis. Nevertheless, the model allows the use of conservation equations (mass, momentum, and energy) to describe the behavior of the fluid. As will be discussed later, the current numerical model solved these conservation equations (one-dimensional) with the associated transport terms represented by classical fluid flow expressions. For example, the shear stress term in the normal-fluid momentum equation was represented by a classical turbulent flow friction correlation, and the convective heat was represented by the enthalpy change between two locations in a duct.

The current numerical model was designed to investigate the forced flow characteristics of He II in a large-diameter transfer line ($d > 1$ cm). Figure 1 shows a schematic of a He II resupply system, in which He II is pumped from the supply tank to the receiver tank via the transfer line. The transfer line and receiver tank were assumed to have been cooled below the lambda temperature prior to the liquid transfer. The transient behavior of the transfer line and receiver tank cooldown is out of the scope of this paper. During the transfer, the liquid flowing inside the cold duct is still subjected to environmental heat loads, such as heat leaks on the transfer line and couplings. Accordingly, the superfluid concentration in the liquid (which decreases as the temperature increases) changes as it flows inside the duct. The amount of enthalpy increase in the liquid depends on the mass flow rate, line size, and the amount of heat absorbed. Since mass flow rate depends on pump capacity

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Presented as Paper 87-1495 at the 22nd AIAA Thermophysics Conference, Honolulu, HI, June 9–11, 1987; received July 21, 1987; revision received May 16, 1988.

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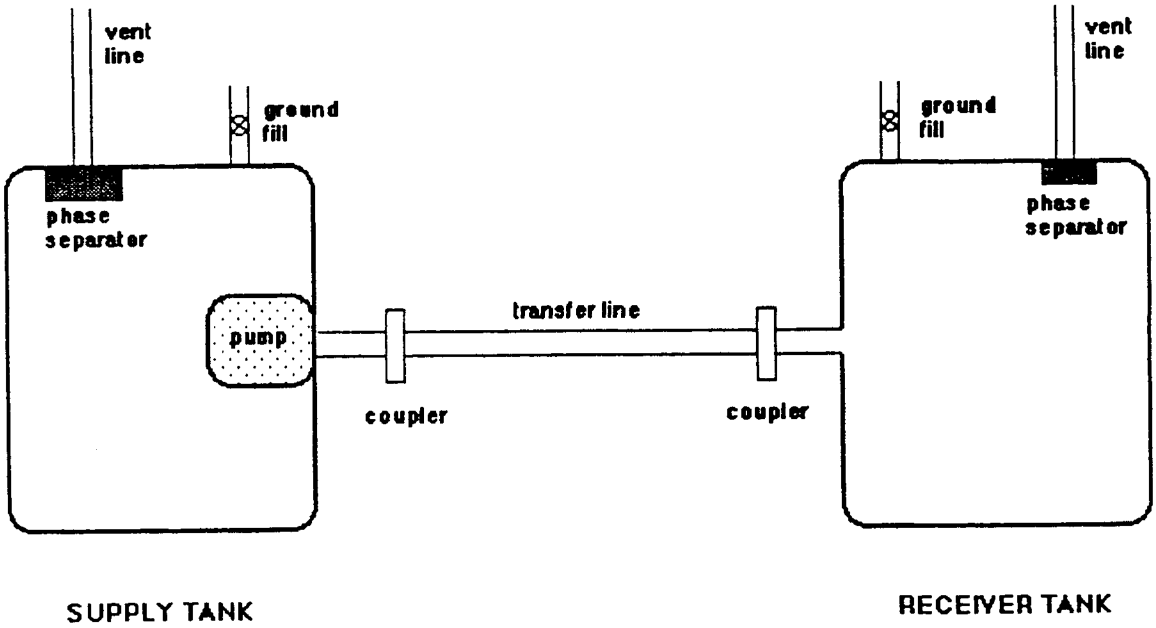

Fig. 1 Schematic of Helium-II transfer system.

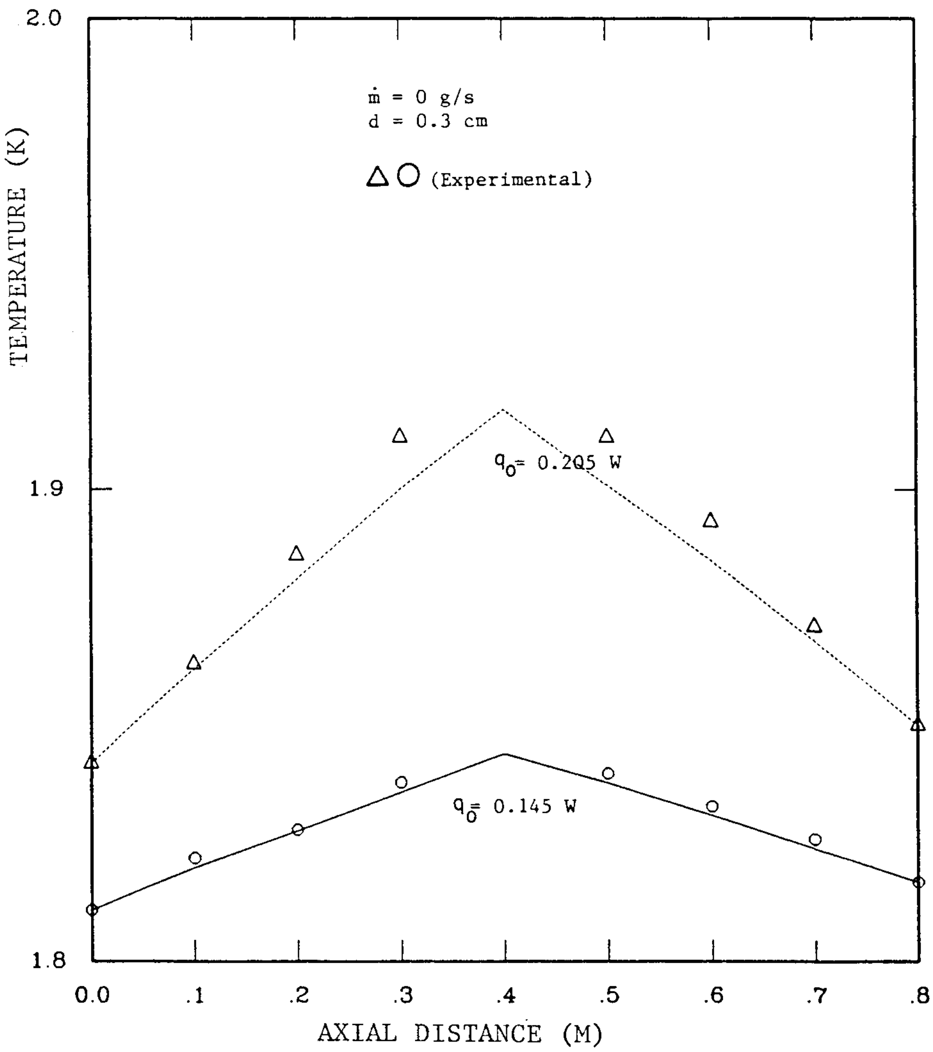

Fig. 2 Comparison of predicted helium temperature with experimental data.[6]

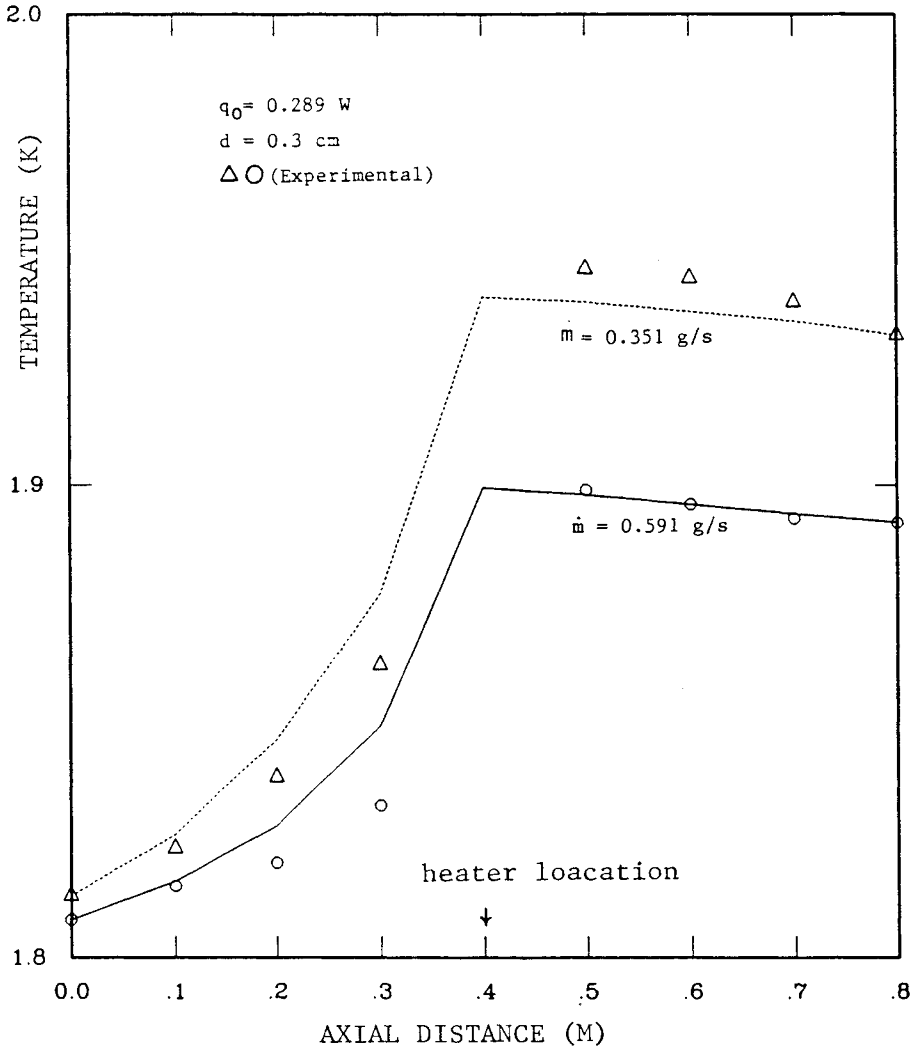

Fig. 3 Comparison of predicted helium temperature with experimental data.[6]

available to drive the flow, it is important to design a pump that can provide a sufficient head to overcome the pressure loss and avoid local boiling in the transfer line.

Modeling of He II under forced flow conditions was performed previously at different levels of complexity. Hermanson et al.[4] developed a model to study superfluid helium flow under the driving head of a thermomechanical pump. The model simulated two extreme cases of the flow: high thermal conductivity laminar flow bound and zero conductivity turbulent flow bound. Kashani and Van Sciver[5,6] (using a bellows-type pump in their experiments) and Srinivasan and Hofmann[7] (using a thermomechanical pump) each developed a heat-transfer model to predict He II temperature data measured in their experiments. However, momentum transfer was not considered in their models. The current model is the first model that directly solves the coupled energy and momentum equations of superfluid and normal fluid to predict the local pressure and fluid temperature along the transfer line. The model can be used for designing large-scale He II transfer systems. It also demonstrates the suitability of using classical flow correlations for predicting the complicated He II flow behavior in large ducts.

The present paper mainly discusses the structure and verification of the computer model. The application of the model to assess the pump requirement for spaceborne systems such as SIRTF and SHOOT is discussed in separate papers.[8,9]

Model Description

The current model basically solved the one-dimensional steady-state conservation equations in the two-fluid model of He II using a finite-difference method. The temperature and pressure dependent thermophysical properties of He II in the current model were obtained from the National Bureau of Standards (NBS). Only turbulent flows ($Re > 10^5$) are analyzed here, as they characterize those flows that typically exist inside a large-diameter duct under forced flow conditions. The model can be easily modified to include laminar flows or flows with lower Reynolds numbers. The input parameters to the model are 1) size of the duct, 2) amount of parasitic heat load on the duct, and 3) inlet mass flow rate and pressure. The output of the model includes He II temperature and pressure distributions, and the two-fluid component velocities. The computation also checks the occurrence of boiling in the transfer line for a given flow rate and parasitic heat load, if the liquid's metastable state is not considered.

The steady-state one-dimensional two-fluid model of He II consists of the following conservation equations:

Continuity

$$\rho U = \rho_s U_s + \rho_n U_n \tag{1}$$

Energy

$$\rho U C_p \frac{\partial T}{\partial z} = \frac{\partial}{\partial z}\left[K\left(\frac{\partial T}{\partial z}\right)^{1/3}\right] + q_w''' + q_0 \delta(z - z_0) \tag{2a}$$

and

$$K = \rho_s S T (S/A_{\mathrm{GM}} \rho_n)^{1/3} \tag{2b}$$

Superfluid component momentum

$$\rho_s U_s \frac{\partial U_s}{\partial z} = -\frac{\rho_s}{\rho}\frac{\partial P}{\partial z} + \rho_s S \frac{\partial T}{\partial z} - F_{\mathrm{mf}} + F_{\mathrm{rel}} \tag{3}$$

Normal component momentum

$$\rho_n U_n \frac{\partial U_n}{\partial z} = -\frac{\rho_n}{\rho}\frac{\partial P}{\partial z} + \rho_s S \frac{\partial T}{\partial z} + \tau_n''' - F_{\mathrm{mf}} + F_{\mathrm{rel}} \tag{4a}$$

In the energy equation, two assumptions[10] have been made: 1) the heat flow caused by the internal convection mechanism (also known as Gorter–Mellink conduction) is not influenced by the net mass flow of He II, and 2) the heat carried by forced convection can be described as the change in enthalpy between two locations, similar to that describing the forced convection for ordinary fluid. The internal convection mechanism in the flow range considered in the present work results from the interaction of vortices of superfluid with normal fluid. The interaction produces a frictional effect called mutual friction F_{mf}. An expression for F_{mf} will be discussed later. The parasitic heat in the energy equation includes the environmental heat load on the transfer line q''' and other discrete heat source q_0 such as couplings or field joints located at z_0.

Two options of boundary conditions for the energy equation are available in the model:

Condition 1

$$T = T_0 \quad \text{at} \quad z = 0$$

$$T = T_L \quad \text{at} \quad z = L \tag{2c}$$

Condition 2

$$T = T_0 \quad \text{at} \quad z = 0$$

$$\frac{\mathrm{d}T}{\mathrm{d}z} = 0 \quad \text{at} \quad z = L \tag{2d}$$

The first boundary condition prescribes fixed temperatures at both ends of the transfer line. The second boundary condition assumes that the temperature of He II at the transfer line exit is identical to that in the receiver tank (adiabatic boundary condition). To investigate the forced convective heat transfer in He II flow, some experimenters[6,7] used the first boundary condition as their operational condition. As will be discussed in the subsequent section, data from these experiments are used to compare with the current model prediction. For large spaceborne transfer systems, in which He II is transferred from a nearly full supply tank to an empty precooled receiver tank, the second boundary condition is more suitable and applicable in analyzing heat transfer.

In the momentum equations, F_{mf} is the mutual friction term and F_{rel} the chemical potential term caused by the two components' relative motion[2,3]

$$F_{\mathrm{mf}} = A_{\mathrm{GM}} \rho_n \rho_s (U_s - U_n)^3 \tag{4b}$$

$$F_{\mathrm{rel}} = \frac{\rho_s \rho_n}{2\rho} \frac{\partial}{\partial z} (U_n - U_s)^2 \tag{4c}$$

A_{GM} is the Gorter–Mellink coefficient of mutual friction and can be expressed in the form

$$A_{\mathrm{GM}}^{-1} = K_{\mathrm{GM}}^3 [(\rho_s/\rho)\mu_n] \tag{4d}$$

where K_{GM} was found to be about 11.3 for turbulent flow.[11,12] The shear stress term of normal fluid τ_n''' is represented by a classical friction coefficient correlation

$$\tau_n''' = -(4/D) f (\rho U_n^2/2) \tag{4e}$$

The Fanning friction coefficient f for a Reynolds number in the range of $10^4 < Re < 5 \times 10^6$ is estimated by[13]

$$f/2 = [2.236 \ln(Re_n) - 4.639]^{-2} \tag{4f}$$

where

$$Re_n = (\rho U_n D/\mu_n) \tag{4g}$$

By combining Eqs. (3) and (4a) and rearranging, the following equation results:

$$\rho_s U_s \frac{\partial U_s}{\partial z} + \rho_n U_n \frac{\partial U_n}{\partial z} = -\frac{\partial P}{\partial z} + \tau_n''' \tag{5}$$

Equations (1–3) and (5) were solved numerically using the second upwind finite-difference method.[14] Equation (5) was solved instead of Eq. (4a) because the former simplifies the calculation. The unknowns of the equations are the He II pressure, temperature, and velocities of the two fluid components.

The finite-difference form of the foregoing conservation equations was written in terms of a staggered grid scheme in the Eulerian mesh. The flow channel was divided into control elements with state variables (pressure and temperature) defined at the center of a node and the flow variable (velocity) defined at the node boundaries. The computer program was written in FORTRAN and can be run on VAX or CRAY.

Results

Model Verification

The accuracy and validity of the model were verified by comparing the predictions with experimental data available in the literature. Experimental data of Kashani and Van Sciver[6] and Srinivasan and Hofman[7] were compared with model predictions. Srinivasan and Hofman used a thermomechanical pump to drive He II flow through a test section of 0.8 m in length and 3 mm in diameter, whereas Kashani and Van Sciver used a bellows-type pump through a test section of 2 m in length and 3 mm in diameter. Both experiments had a heater installed in the middle of the test section. The upstream and downstream bath temperatures were kept constant throughout all test runs. This operational condition is equivalent to the boundary condition given by Eq. (2c).

Figures 2–4 compare the predicted local He II temperature with Srinivasan's data. Figure 2 shows the results for runs conducted with the pump turned off (or $\dot{m} = 0$ g/s). When the pump is turned off, no net flow occurs and the heat transfer

Table 1 Comparison of calculated friction coefficient (Darcy) with experimental data[11] (tube roughness[19] = 1.5×10^{-3} mm, curved tube, $T_{\mathrm{He}} = 1.8$ K)

Re	$C_{f,\mathrm{measured}}$	$C_{f,\mathrm{model}}$
1.30×10^5	0.0232	0.0252
1.64×10^5	0.0231	0.0246
2.31×10^5	0.0227	0.0238
2.94×10^5	0.0223	0.0235

Table 2 Comparison of calculated friction coefficient (Fanning) with experimental data[16] (smooth tube, $T_{\mathrm{He}} = 1.8$ K)

Re	f_{measured}	f_{model}
9.00×10^5	0.00310	0.00295
1.25×10^6	0.00280	0.00279
1.50×10^6	0.00275	0.00271
1.75×10^6	0.00275	0.00264

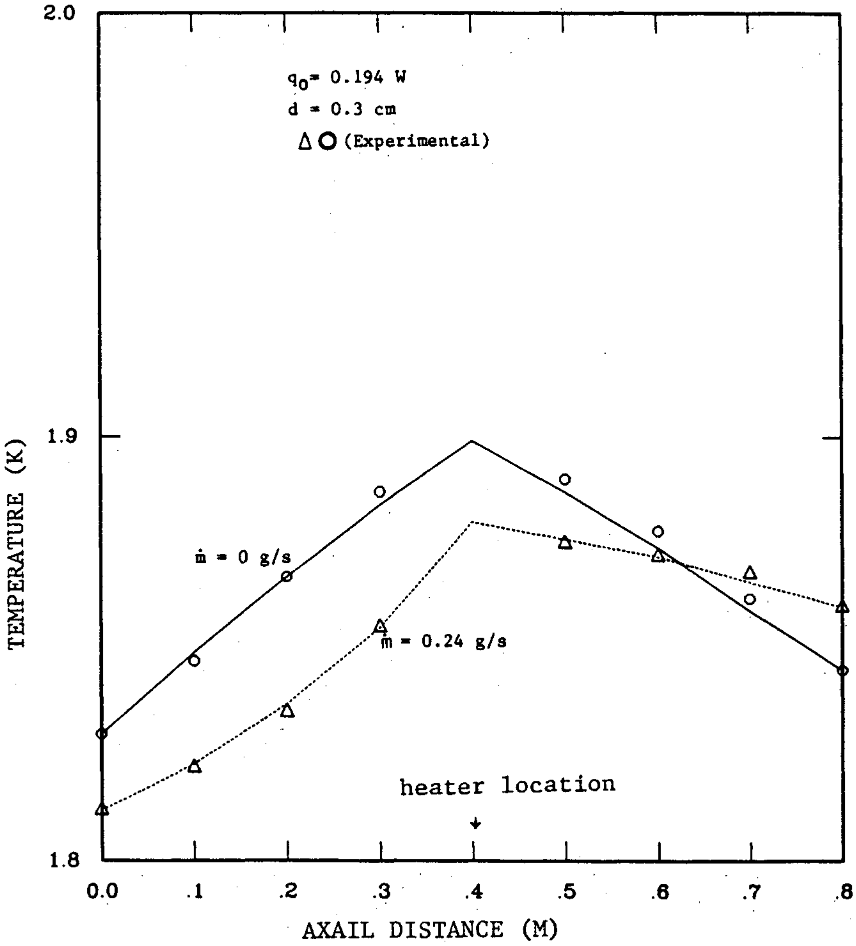

Fig. 4 Comparison of predicted helium temperature with experimental data.[6]

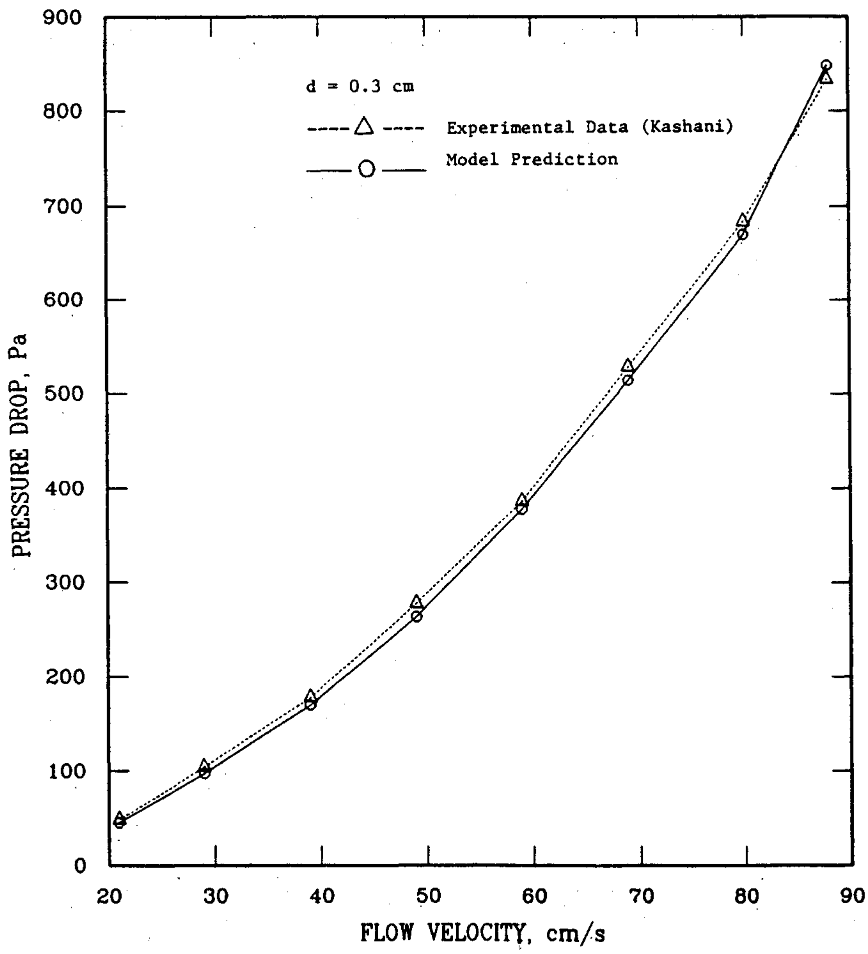

Fig. 5 Comparison of predicted pressure drop with experimental data.

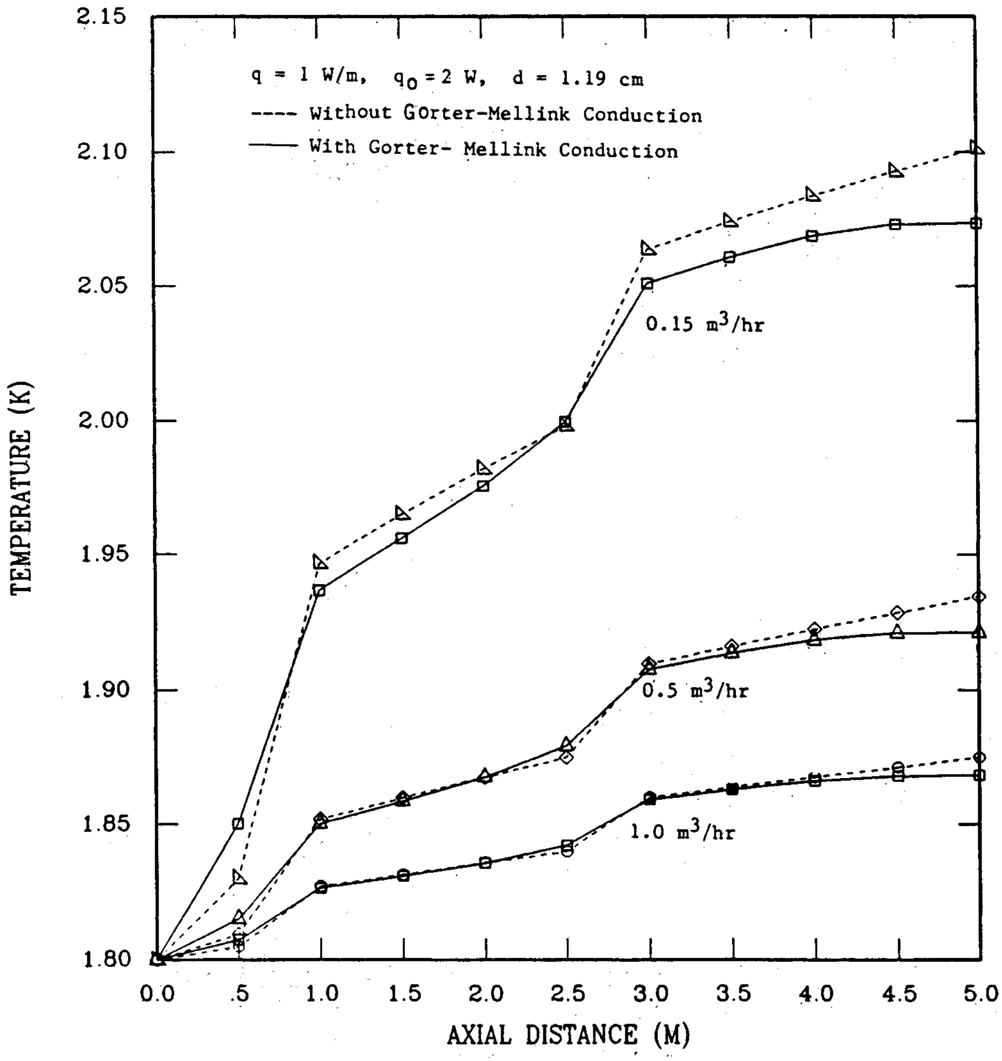

Fig. 6 Predicted Helium-II temperature for different flow rates.

is solely by means of the Gorter–Mellink conduction (internal convection). The temperature distribution is observed to be quite symmetrical about the heater location. The predictions agree well with the experimental results. This implies that the Gorter–Mellink conduction term used in the energy equation [Eq. (2a)] describes quite accurately the underlying heat-transfer mechanism for no net He II flow. The results for runs conducted with the pump operating are shown in Fig. 3. The effect of forced convection on temperature distribution is clearly illustrated. Net flow carries more heat downstream from the heater. The predictions again agree quite well with the measured data. This indicates that the classical expression of forced convection may sufficiently describe the forced convective heat-transfer mechanism in the He II flow. Figure 4 compares the prediction with experimental data for a different heat load on the transfer line. The predictions are also in good agreement with empirical data. For all of the predictions made, the largest discrepancy from empirical data is about 30%. The discrepancy could be caused by several factors such as the validity of the assumptions made in modeling the energy transport [e.g., the classical forced convective term used in Eq. (2a)], numerical error, and possibly experimental errors.

The comparison between predicted and measured pressure drops across the transfer line is shown in Fig. 5. The experimental data were taken from Kashani and Van Sciver.[6] The Reynolds number of the flow considered in these tests ranged from 10^4 to 3×10^5, indicating the flow was in the turbulent regime. As illustrated in the figure, the predicted pressure drops agree well with experimental data. The maximum discrepancy between predictions and measurement is about 6%. The friction coefficients used in the model calculation [Eq. (4f)] were also compared with the measured results. Table 1 compares the Darcy friction coefficient for a rough curve duct, and Table 2 the Fanning friction coefficient for a straight duct. Again, the calculated and measured data agree well.

The foregoing comparisons indicate that the current model is capable of predicting He II flow behavior in small-scale experiments. It is expected that the model can be used in the design of large-scale spaceborne transfer systems. The model not only aids in design purposes, but also provides some insight into the physical mechanism existing in turbulent He II flow inside a large duct.

Classical Turbulent Flow Approximation

Experiments[5,17] showed that at high flow rates ($Re > 10^5$) forced convection usually dominates over the Gorter–Mellink conduction in He II heat transfer. This behavior was also observed in the current model predictions. Several test runs were made to examine the significance of Gorter–Mellink conduction on heat transfer in the He II flow. In these runs, the transfer line was 5 m in length and 1.19 cm in diameter and had two couplings, each with a conservative heat leak of 2 W. Two cases were examined for each of the following flow rates: 0.15 m^3/h, 0.5 m^3/h, and 1.0 m^3/h. The first case considered both the Gorter–Mellink conduction and forced convection in the temperature calculation with the boundary condition given

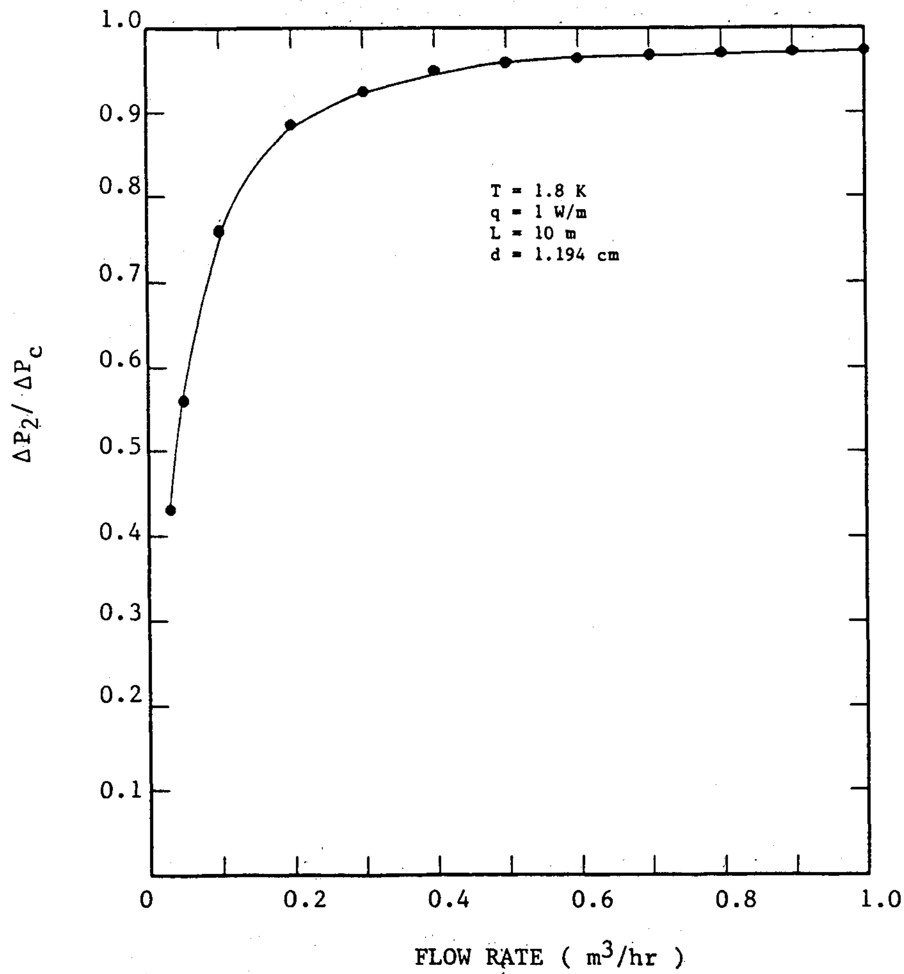

Fig. 7 Comparison of pressure drops predicted by two-fluid model with those predicted by classical turbulent flow correlation.

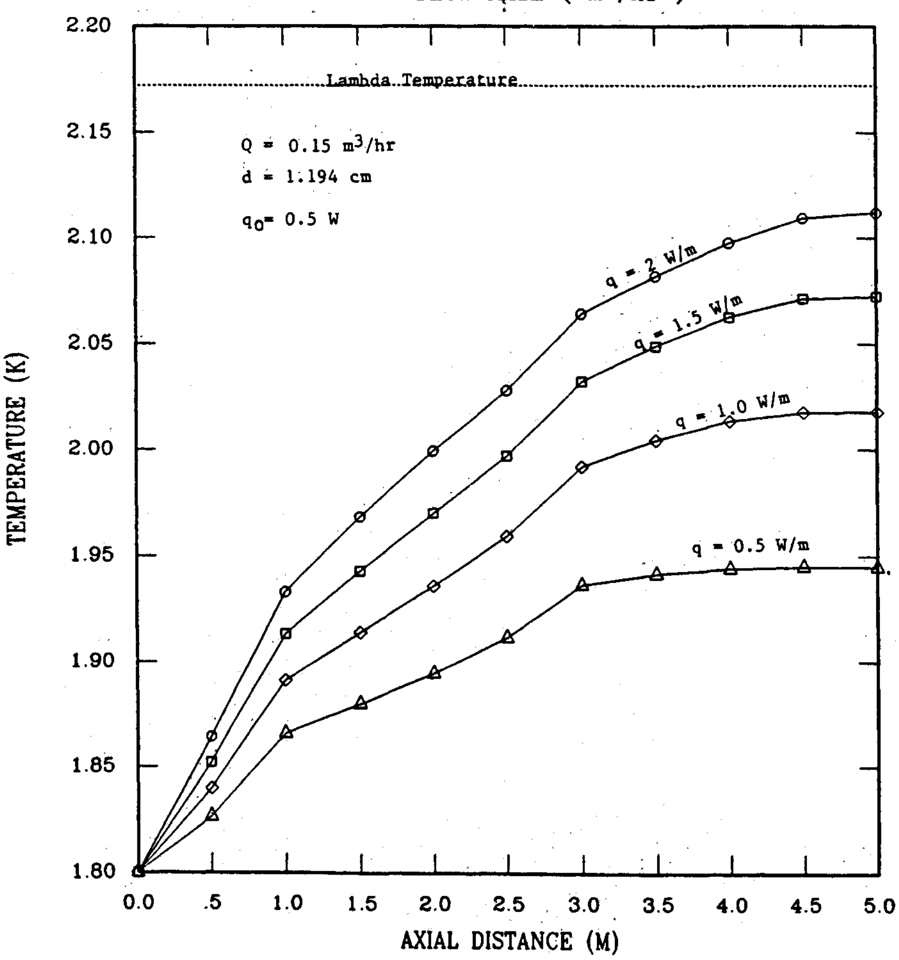

Fig. 8 Predicted Helium-II temperature with different heat loads on the transfer line.

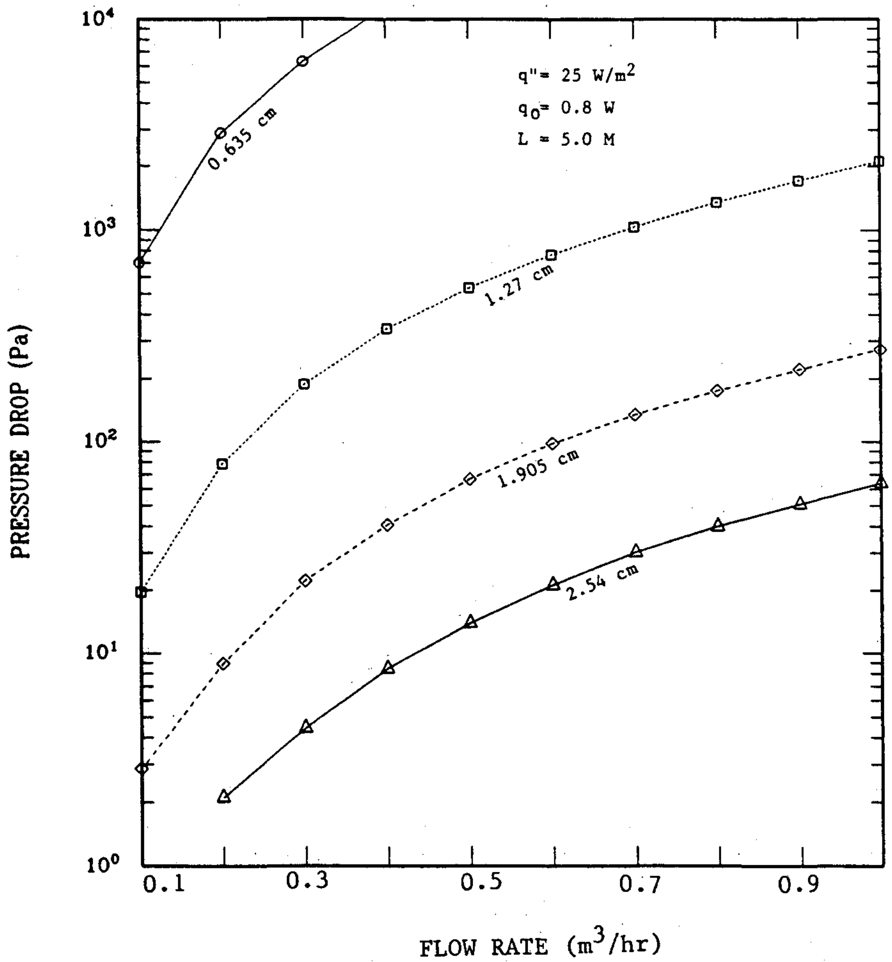

Fig. 9 Predicted pressure drop across transfer line for different diameters.

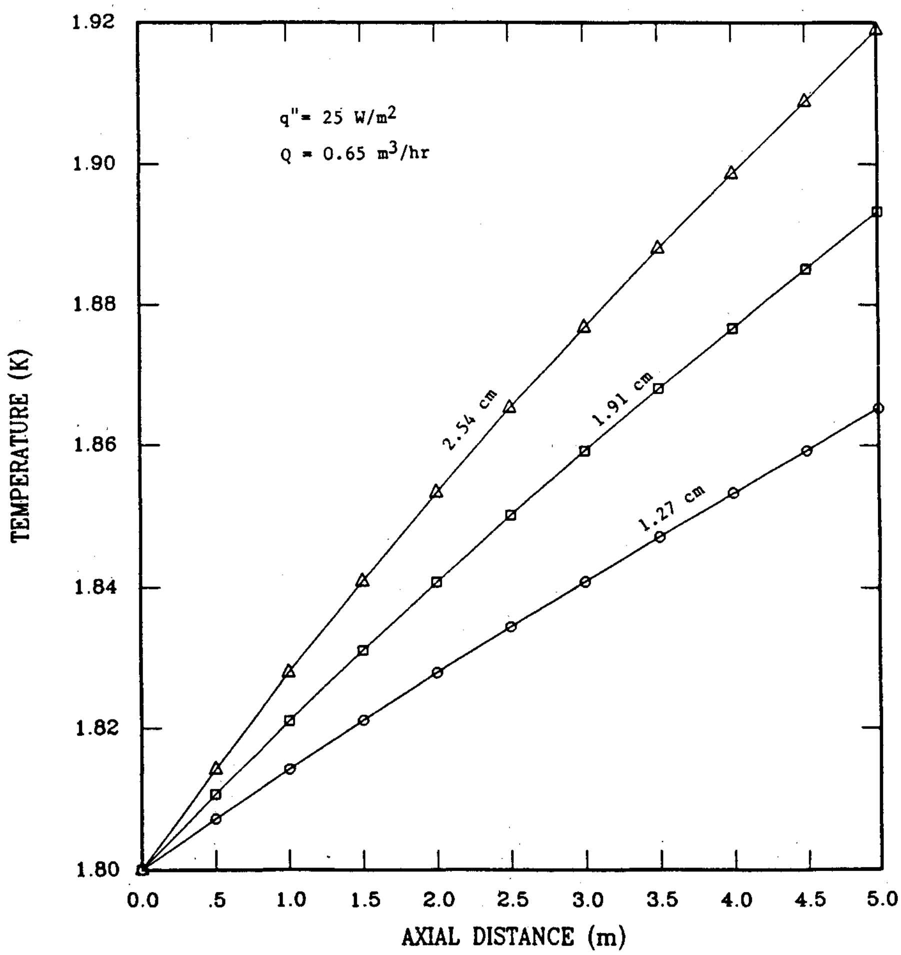

Fig. 10 Predicted Helium-II temperature in transfer line for different diameters.

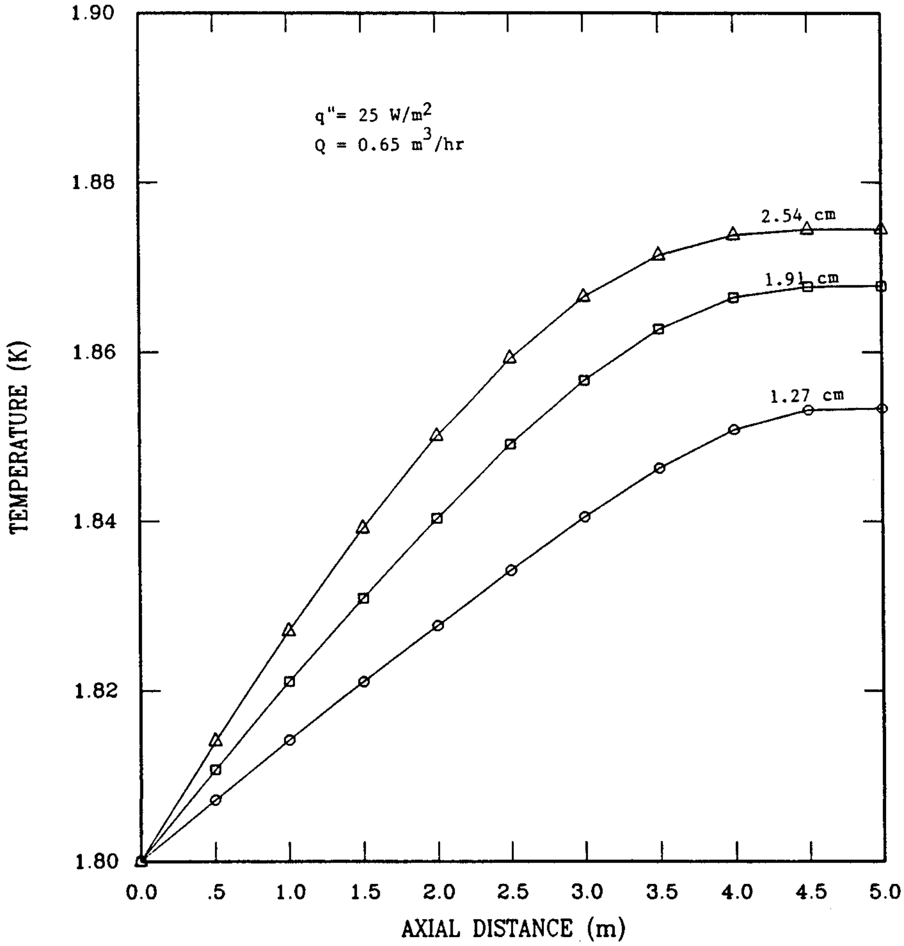

Fig. 11 Predicted Helium-II temperature in transfer line for different diameters (with Gorter-Mellink conduction).

by Eq. (2d). The second case considered only forced convection, and no boundary condition is necessary. Flow rates greater than 0.15 m^3/h were examined because this range is typically desirable for spaceborne helium transfer. The predicted fluid temperature for the two cases was superimposed in Fig. 6 for direct comparison. The temperature gradient near the inlet is higher for the first case, indicating that more heat is conducted back to the upstream (supply tank), resulting in a lower overall temperature increase at the exit of the duct. The trend is more apparent for low flow rates. As the flow rate increases, the difference in fluid temperature at the exit between the two cases is insignificant, e.g., about 10% difference for 0.15 m^3/h, 5% for 0.5 m^3/h, and 2% for 1.0 m^3/h. Thus, at high flow rates, forced convection dominates over the Gorter-Mellink conduction, and the latter can be neglected in He II heat-transfer analysis.

Experiments also showed that at high transfer rates, He II behaved like a classical turbulent flow. Figure 7 compares the pressure drop across the transfer line calculated from the two-fluid model (ΔP_2) with that calculated from the classical turbulent flow (ΔP_c) expression given by

$$\Delta P_c = 4f(\rho U^2/2)(L/D) \qquad (6)$$

where f was determined by Eq. (4f). The pressure drop ratio ($\Delta P_2/\Delta P_c$) is plotted vs flow rate in the figure. At flow rates below 0.1 m^3/h ($Re < 3 \times 10^5$), ΔP_2 differs quite considerably from ΔP_c. When the flow rate exceeds 0.2 m^3/h, the discrepancy between ΔP_2 and ΔP_c is less than about 10%. As flow rate increases further, the two-fluid model prediction approaches the classical flow prediction. This may be explained as follows. In the two-fluid model, fluid acceleration, wall shear stress, mutual friction, and the thermomechanical force in the superfluid and normal fluid momentum equations contributed to the overall pressure drop. Under forced flow conditions, mutual friction usually counteracts the thermomechanical force. At high flow rates, the fluid acceleration and difference between mutual friction and the thermomechanical force are small compared to the wall shear stress. Consequently, the wall shear stress dominates the pressure drop, resulting in ΔP_2 approaching ΔP_c. At low flow rates, the shear stress no longer dominates the pressure loss, and all the other terms in the momentum equations of the two-fluid model become significant. Hence, significant deviation from the classical flow solution may occur.

Parametric Study

The current model predicts pressure drop and He II temperature distribution in a transfer line. It can be used to design large-scale He II transfer systems (such as to assess pump requirements and design a thermally efficient transfer line). In this section, the effects of parasitic heat load and transfer line size on He II heat transfer will be discussed.

Thermal insulation plays an important role in determining the amount of parasitic heat transmitted into the transfer line. Poor thermal insulation, which causes a higher parasitic heat load, may result in converting superfluid into normal fluid, or even liquid boiling in the transfer line. Figure 8 shows the predicted temperature profile of He II for various parasitic heat loads. In calculating the temperature distribution, the boundary condition given by Eq. (2d) was used.

It is also essential to design a pump that will be able to provide sufficient head to drive the He II through the transfer line without causing any local boiling of the liquid. The predicted pressure drop across a hypothetical transfer line for different duct diameters is illustrated in Fig. 9. It is obvious that for a smaller duct size, a higher pump head is required to drive He II at a given flow rate. At the time this paper was written, the maximum pump head available for He II transfer was about 170 Torr (2.23×10^4 Pa) for the NBS centrifugal pump,[15] and 260 Torr (3.41×10^4 Pa) for the NASA Goddard thermomechanical pump.[18] The diameter size of the transfer line also can affect the liquid temperature distribution. For a larger duct, because of a larger surface area and a higher parasitic heat load, the increase in liquid temperature across the transfer line is larger. Figure 10 shows the liquid temperature profiles for various diameters of transfer lines. The runs were performed without the Gorter–Mellink conduction in the energy equation. The results in which the Gorter–Mellink conduction has been considered are demonstrated in Fig. 11.

Conclusions

The current model demonstrates the suitability of imposing the "classical flow" lines of thought onto the two-fluid representation of He II for investigating the turbulent He II flow behavior in a large-diameter transfer line. The model predictions agree reasonably well with experimental measurement. At high flow rates (>0.5 m^3/h or $Re > 1.5 \times 10^6$), forced convection generally dominated the heat transfer, and wall shear stress dominated the pressure drop across the transfer line. Accordingly, for turbulent He II flow, it is adequate to analyze pressure drop using classical frictional loss correlation and heat transfer using solely forced convection. The present model simulation is focused on He II flow inside a transfer line and has not considered the thermodynamics of supply and receiver tanks. It is recommended that the tanks' thermodynamics should be included in the model for an end-to-end simulation of He II transfer.

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